
THE ESSENTIAL TOOL

2.0	What We Need to Know When We Finish This Chapter	35
2.1	Is This Really a Math Course in Disguise?	37
2.2	Fun with Summations	38
2.3	Constants in Summations	41
2.4	Averages	43
2.5	Summations of Sums	44
2.6	More Fun with Summations of Sums	48
2.7	Summations of Products	52
2.8	Time to Reflect	53
	Exercises	54

2.0 What We Need to Know When We Finish This Chapter

This chapter reassures us that we can handle the material in this course. It reviews the fundamental results regarding the summation, the principal algebraic tool. Here are the essentials.

1. **Section 2.1:** This is not a math course. This is almost all just addition, subtraction, multiplication, and division. We can do this.

2. **Equation (2.5), section 2.3:** The summation of a constant is n times that constant:

$$\sum_{i=1}^n a = na.$$

3. **Equation (2.7), section 2.3:** Constants factor out of summations:

$$\sum_{i=1}^n ax_i = a \sum_{i=1}^n x_i.$$

Variables do not:

$$\sum_{i=1}^n ax_i \neq x_i \sum_{i=1}^n a.$$

4. **Equation (2.8), section 2.4:** The average of the x_i 's is

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}.$$

5. **Equations (2.9) and (2.10), section 2.4:** Weighted averages are

$$\bar{x}_w = \frac{\sum_{i=1}^n a_i x_i}{n},$$

where

$$\sum_{i=1}^n a_i = n.$$

6. **Equation (2.14), section 2.5:** The summation of a sum is the sum of the individual summations:

$$\sum_{i=1}^n (x_i + y_i) = \sum_{i=1}^n x_i + \sum_{i=1}^n y_i.$$

7. **Equation (2.19), section 2.5:** The sum of the deviations from the average is zero:

$$\sum_{i=1}^n (x_i - \bar{x}) = 0.$$

8. **Equation (2.21), section 2.6:**

$$\sum_{i=1}^n (x_i - \bar{x}) \bar{x} = 0.$$

9. **Equation (2.28), section 2.6:**

$$\sum_{i=1}^n (x_i - \bar{x}) x_i = \sum_{i=1}^n (x_i - \bar{x})^2.$$

10. **Equation (2.37), section 2.6:**

$$\sum_{i=1}^n (x_i - \bar{x}) y_i = \sum_{i=1}^n (x_i - \bar{x}) (y_i - \bar{y}) = \sum_{i=1}^n (y_i - \bar{y}) x_i.$$

11. **Equation (2.40), section 2.7:** Products within summations can be distributed and summed individually:

$$\sum_{i=1}^n (x_i + y_i) z_i = \sum_{i=1}^n x_i z_i + \sum_{i=1}^n y_i z_i.$$

That's it! There are lots of other equations in this chapter, but they're all here either to help derive or to help understand those listed here.